AN EVEN-PHASE MULTI-MOTOR VECTOR CONTROLLED DRIVE WITH SINGLE INVERTER SUPPLY AND SERIES CONNECTION OF STATOR WINDINGS

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ABSTRACT: Vector control principles enable independent flux and torque control of an ac machine by means of only two stator d-q axis current components. This means that in ac machines with a phase number greater than three there exist additional degrees of freedom, which are nowadays used to enhance the overall torque production of a multi-phase machine through injection of higher stator current harmonics. However, these additional degrees of freedom can be used to control independently other machines within a multi-motor drive system. In order to do so, it is necessary to connect in series stator windings of all the multi-phase machines, with an appropriate phase transposition. A vector control algorithm is then applied to each machine separately, total inverter phase current references are created by summation of individual machine phase current references and supply to the stator windings of the multi-machine set is provided from a single current controlled voltage source inverter (VSI). The concept is introduced in this paper using the general theory of electrical machines and all the possible situations that may arise for an even number of phases are examined. Although an induction motor drive is considered throughout, the concept is equally applicable to all ac machines with sinusoidal stator and rotor flux distribution. Its main advantage is the potential for saving in the number of inverter legs, compared to an equivalent three-phase motor drive system. The saving depends on the number of phases and, for an even phase number, comes into existence when the number of phases is equal to or greater than eight. Various even phase numbers are considered in detail and appropriate connection diagrams are given. Verification of the proposed multi-phase multi-motor drive is provided by simulation of a ten-phase four-motor system.

1. INTRODUCTION

Industrial electrical drive applications often require a number of variable speed drives. Examples include textile and paper manufacturing, robotics, traction, electric vehicles, etc. In vast majority of cases these multi-motor drive systems require independent control of individual motors. Standard solution in such situations is to use a set of three-phase motors, which are supplied from their own PWM VSI and all the inverters are connected to the common dc link. Either vector control or direct torque control can be used for independent control of the motors within a group in order to obtain a high performance [1]. If a multi-motor system consists of \( k \) three-phase machines, this approach requires \( 3k \) inverter legs. Numerous solutions have therefore been proposed for reducing the total number of inverter legs required in a multi-machine system [2-4]. The two solutions analysed in detail in [4] require only \( 2k \) and \( 2k+1 \) inverter legs, respectively. However, both configurations lead to a substantial increase in the total harmonic distortion and to a reduced voltage capability. The authors conclude that the configuration with \( 2k+1 \) legs offers a better performance than the configuration with \( 2k \) legs. Both are however inferior with respect to the standard solution with \( 3k \) legs.
In certain applications, such as for example traction, it is possible to use a multi-motor system that consists of \( k \) three-phase motors connected in parallel and supplied from a single PWM VSI [5-7]. Vector control is used in this case and it is a prerequisite that all the motors develop an identical electromagnetic torque [7]. Due to parallel connection and supply from a single inverter, the whole multi-machine system always has to operate with the same supply voltage and frequency, so that the means for independent control of individual motors within the multi-machine system do not exist. Very much the same applies to another parallel connection of \( k \) three-phase motors, supplied this time from a single PWM current source inverter [8]. Since V/f control is used in [8], the motors are always subjected to the same supply voltage and frequency conditions.

Supply of variable speed electric drives from inverters enables substitution of the standard three-phase configuration with an appropriate multi-phase (\( n \)-phase) configuration. Ever since the inception of the first multi-phase (five-phase) inverter-fed induction motor drive in 1969 [9], the interest in multi-phase drive systems has been steadily increasing, as evidenced by recent survey papers [10,11]. Major advantages of using a multi-phase machine instead of a three-phase machine are detailed in [12] and can be summarised as follows: higher torque density, greater efficiency, reduced torque pulsation, greater fault tolerance, reduction in the required rating per inverter leg and therefore simpler, more reliable power conditioning equipment. Additionally, noise characteristics of the drive improve as well [13]. Higher torque density in a multi-phase machine is possible since, apart from the fundamental spatial field harmonic, space harmonic fields can be used to contribute to the total torque production [12,14-17]. This advantage stems from the fact that vector control of the machine’s flux and torque, produced by the interaction of the fundamental field component and the fundamental stator current component, requires only two stator currents (d-q current components). In a multi-phase machine, with at least five phases or more, there are therefore additional degrees of freedom, which can be utilised to enhance the overall motor torque production through injection of higher order stator current harmonics. The concept is equally applicable to any ac machine type and has been successfully demonstrated for induction [15,17] and synchronous reluctance [14,16] machines. In a five-phase machine the third harmonic current injection can be used [14,15,17], while in a nine-phase machine it is possible to use injection of the third, the fifth and the seventh current harmonic [16]. This is so since injection of any specific current harmonic requires again two current components, similar to the torque/flux production due to fundamental harmonic. In general, the possibility for an increase in the torque density increases as the number of phases increases. However, it appears that once the number of phases reaches fifteen, further increase does not provide any further important advantage [12].

As already noted, vector control of a multi-phase machine requires only two currents if only the fundamental of the field is utilised. Although the remaining degrees of freedom can be used to enhance the overall motor torque production, they can also be used in an entirely different manner, as the basis for a multi-motor multi-phase drive system development. This papers attempts to develop this idea in a systematic manner. An \( n \)-phase ac machine is considered, such that the number of phases is an even number. It is shown that, by connecting the multi-phase stator windings of the \( k \) machines in series, with an appropriate phase transposition, it becomes possible to realise a completely independent vector control of the machines although only one inverter is used. There are not any restrictions on the type of the ac machine used within the drive system, machines’ power rating, loading and operating speed. The initial idea behind this concept was for the first time indicated in [18], where the notion of an \( n \)-dimensional space for an \( n \)-phase machine [19] was applied in the analysis and only a two-motor, five-phase system was studied. The concept has never been examined for an even number of phases and is developed here
using the general theory of electric machines [20]. The necessary phase transposition in the series winding connection is established by analysing the properties of the decoupling transformation matrix. So-called connectivity matrix is introduced and connection diagrams are given for some characteristic phase numbers. Classification of all the even phase numbers is further provided, with regard to the maximum number of connectable machines and the number of phases of individual machines in the group. The concept is verified by simulation of a ten-phase four-motor drive system in torque mode of operation. Its main advantages and shortcomings are assessed in the concluding section of the paper. A corresponding study for an arbitrary odd number of phases has also been done and the results will be reported in near future.

**2. MODELLING OF AN n-PHASE AC MACHINE**

**2.1 General remarks**

An $n$-phase machine, such that the spatial displacement between any two consecutive stator phases equals $\alpha = \frac{2\pi}{n}$, is under consideration. Although the type of the ac machine is irrelevant, it is assumed that the machine is an induction motor. Both stator and rotor windings are treated as $n$-phase, for the sake of generality. It is assumed that the spatial distribution of the stator and rotor flux is sinusoidal, since the intention is to control torque production due to fundamental harmonic only. All the inductances within the stator and the rotor $n$-phase winding are therefore constants and the mutual inductances between the stator and rotor phases contain only the fundamental harmonic. Rotor winding is taken as referred to the stator winding. All the other standard assumptions of the general theory of electrical machines apply. The phase number can be odd or even. Considerations are in this paper restricted to an even number of phases.

It is important to note that only the true $n$-phase machines are discussed here. This means that the $m$-th phase and the $(n/2 + m)$-th phase are positioned 180 degrees apart and are supplied with currents in phase opposition. Each such an $n$-phase winding can be reconfigured into so-called semi $n$-phase winding [10], with an effective number of phases equal to $n/2$ (the customary three-phase machine is in essence a semi six-phase machine, [10]). While this reconfiguration halves the number of phases and would therefore halve the number of inverter legs, it would simultaneously halve the number of connectable machines as well. Furthermore, if the original even phase number $n$ is such that $n/2$ is a prime number, semi $n$-phase winding becomes a winding with an odd number of phases (these phase numbers are, as already noted, beyond the scope of this paper). If the original phase number $n$ is a power of two, reconfiguration into a semi $n$-phase winding requires a neutral conductor for operation with the balanced system of currents [10] (a semi four-phase machine is customarily known as a two-phase machine; since the currents are 90 degrees apart, the two-phase system requires three wires). It is for these reasons that the analysis in the paper is restricted to true $n$-phase systems ($n = \text{even}$), with spatial displacement between any two consecutive windings equal to $\alpha = \frac{2\pi}{n}$.

**2.2 Decoupling transformation**

The machine model in phase variable form is transformed using decoupling (Clark’s) transformation matrix [20]. Decoupling transformation substitutes the original set of $n$ phase currents with a new set of $n$ transformed currents. Decoupling transformation matrix for an arbitrary even phase number can be given in power invariant form with [20]:

```plaintext
\text{Decoupling transformation matrix} 
```
The first two rows in (1) define stator current components that will lead to fundamental flux and torque production \((\alpha - \beta)\) components; stator to rotor coupling appears only in the equations for \(\alpha - \beta\) components), while the last two rows define the two zero sequence components. In between, there are \((n-4)/2\) pairs of rows which define \((n-4)/2\) pairs of stator current components, termed further on x-y components. As will be shown shortly, x-y pairs of current components will play an important role in realising independent control of the machines connected in series.

Equations for pairs of x-y components are completely decoupled from all the other components and stator to rotor coupling does not appear either [20]. These components do not contribute to torque production when sinusoidal distribution of the flux around the air-gap is assumed. Zero sequence components will not exist in any star-connected multi-phase system without neutral conductor. This means that at most \((n-2)/2\) machines (for an even phase number) can be connected in series. In order to do so it is necessary to ensure that the flux/torque producing currents of one machine do not produce flux and torque in all the other machines of the group. A simple series connection of stator windings is obviously not going to achieve this goal. An appropriate phase transposition is therefore required when connecting the windings in series, as discussed shortly.

### 2.3 Rotational transformation

Since stator to rotor coupling appears only in the equations for \(\alpha - \beta\) components regardless of the machine type and torque production due to the fundamental field component is therefore entirely governed by \(\alpha - \beta\) components, rotational transformation is applied to \(\alpha - \beta\) equations only [20]. Assuming transformation into an arbitrary common reference frame \((\theta_s = \int \omega_s dt)\), the transformation matrices for stator and rotor variables are

\[
D_s = \begin{bmatrix}
\cos \theta_s & \sin \theta_s \\
-\sin \theta_s & \cos \theta_s \\
1 & \ldots \\
\end{bmatrix}
\]

\[
D_r = \begin{bmatrix}
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta \\
1 & \ldots \\
\end{bmatrix}
\]

where \(\beta = \theta_r - \theta\) and \(\theta\) is the instantaneous rotor angular position.

Upon application of the decoupling transformation (1) and rotational transformation (2), an \(n\)-phase induction machine is described with the following voltage equilibrium and flux linkage equations \((p = d/dt)\):

\[
C = \sqrt{\frac{2}{n}} \begin{bmatrix}
\mathbf{x}_{n-4} \\
\mathbf{y}_{n-4} \\
0 \\
0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & \cos \frac{n-2}{2} & \cos \frac{n-2}{2} & \ldots \\
\sin \frac{n-2}{2} & \sin \frac{n-2}{2} & \sin \frac{n-2}{2} & \ldots \\
-1 & -1 & -1 & \ldots \\
0 & \sqrt{\frac{2}{n}} & \sqrt{\frac{2}{n}} & \sqrt{\frac{2}{n}} & \ldots \\
0 & \sqrt{\frac{2}{n}} & -\sqrt{\frac{2}{n}} & \sqrt{\frac{2}{n}} & \ldots \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cos \frac{n-2}{2} & \cos \frac{n-2}{2} & \ldots \\
\sin \frac{n-2}{2} & \sin \frac{n-2}{2} & \ldots \\
-\sin \frac{n-2}{2} & -\sin \frac{n-2}{2} & \ldots \\
1 & 1 & \ldots \\
\sqrt{\frac{2}{n}} & -\sqrt{\frac{2}{n}} & \ldots \\
\sqrt{\frac{2}{n}} & \sqrt{\frac{2}{n}} & \ldots \\
\end{bmatrix}
\]
where \( L_m = (n/2)M \) and \( M \) is the maximum value of the stator to rotor mutual inductances in the phase variable model. Torque equation is given with

\[
T_e = PL_m \left[ i_{dr} i_{qr} - i_{dq} i_{dq} \right]
\] (5)

### 3. SERIES CONNECTION OF MULTI-PHASE STATOR WINDINGS

Since only one pair of stator \( \alpha-\beta \) (d-q) current components is required for the flux and torque control in one machine, there is a possibility of using the remaining degrees of freedom ([\((n-2)/2-1\)] pairs of stator x-y current components) for control of other machines that are to be connected in series with the first machine. However, if the control of the machines with series connected stator windings is to be decoupled one from the other, it is necessary that the flux/torque producing currents of one machine do not produce flux and torque in all the other machines in the group. In other words, the connection of stator windings of \( k=\frac{(n-2)}{2} \) multi-phase machines must be such that what one machine sees as the d-q axis stator current components the other machines see as x-y current components, and vice versa. It then becomes possible to completely independently control speed (position, torque) of \( \frac{(n-2)}{2} \) machines while supplying the drive system from a single current-controlled voltage source inverter. Achievement of the stated goal requires an appropriate phase transposition when connecting the stator windings in series. Phase transposition means shift in connection of the phases 1, 2, ..., \( n \) of the first machine to the phases 1, 2, ..., \( n \) of the second machine, etc., where 1, 2, 3, ..., \( n \) is the flux/torque producing phase sequence of the given machine according to the spatial distribution of the phases within the stator winding. The required phase transposition follows directly from the decoupling transformation matrix (1).

According to (1), phase ‘1’ of all the machines will be connected directly in series (the first column in (1)). The phase transposition for phase ‘1’ is therefore 0 degrees and the phase step is zero. However, phase ‘2’ of the first machine will be connected to phase ‘3’ of the second machine, which will be further connected to phase ‘4’ of
the third machine and so on. The phase transposition moving from one machine to the other is the spatial angle \(\alpha\) and the phase step is 1. This follows from the second column of the transformation matrix. In a similar manner phase ‘3’ of the first machine (the third element in the first row of (1) with spatial displacement of 2\(\alpha\)) is connected to the phase ‘5’ of the second machine, which further gets connected to phase ‘7’ of the third machine, and so on. The phase transposition is 2\(\alpha\), and the phase step is 2. This follows from the third column of (1). Further, phase ‘4’ the first machine needs to be connected to the phase ‘7’ of the second machine which gets connected to the phase ‘10’ of the third machine and so on. Here the phase step is 3 and the phases are transposed by 3\(\alpha\). This corresponds to the fourth column in (1). For phase ‘5’ of the first machine the phase transposition equals 4\(\alpha\) and phase step is 4, for phase ‘6’ the phase step is 5 and phase transposition equals 5\(\alpha\), and so on. If a number obtained in this way is greater than the number of phases \(n\), resetting is performed by deducting \(j \times n\) \((j = 1, 2, 3...)\) from the number so that the resulting number belongs to the set \([1, n]\). It should be noted that for an \(n\)-phase supply system \((n = \text{even})\) not all of the \(k\)-connectable machines will be \(n\)-phase. This means that for some machines in the group the resulting set of the utilised phase numbers in the connection diagram, obtained in the described way, will be a sub-set of the set \([1, n]\). The statement is clarified in what follows, by examining certain phase numbers in more detail.

This explanation enables construction of a connection table, which is further called connectivity matrix. Connectivity matrix and the corresponding connection diagram are given next for some selected even phase numbers. Flux/torque producing phase sequence for any particular machine is denoted in the connectivity matrices with numbers 1,2,3\(\ldots\) \(n\) while the notation \(a,b,c,d,\ldots\) is used in corresponding connection diagrams.

The minimum even number of phases that will enable series connection is \(n = 6\). The corresponding connectivity matrix, obtained on the basis of the given procedure, is given in Table I. As can be seen from the last row of this matrix, only phases 1,3 and 5 of the second machine are utilised. As spatial displacement between these phases is 120\(^\circ\), it follows that the second machine is a three-phase machine rather than a six-phase machine. The corresponding connection diagram is given in Fig. 1. Note that the flux/torque producing currents of the six-phase machine mutually cancel at the connection points with the three-phase machine. This means that the three-phase machine will not suffer from any adverse effects due to the series connection with the six-phase machine. This of course does not hold true for the six-phase machine.

In the case of an eight-phase system it is possible to connect three machines in series. The connectivity matrix is the one given in Table II. The first and the third machine are eight-phase, however the second machine is four-phase since only phases 1,3,5,7 are utilised (and spatial displacement is therefore 90 degrees). It is important to note that, when connecting the machines in series to the source, all the machines with the highest phase number must come first. This means that the actual sequence of connection of the three machines to the source has to be M1, M3, M2, as shown in the connection diagram in Fig. 2. This is so since flux/torque producing currents of the machine with a higher phase number cancel when entering the machine with the lower phase number (for example, in the six-phase case of Fig. 1. phase currents \(a1\) and \(d1\) of the six-phase machine are in phase opposition, so that their sum at the point of entry into the phase \(a2\) of the second machine is zero).

Ten-phase case is illustrated in Table III and Fig. 3. It is now possible to connect four machines in series. Two of them are ten-phase (M1 and M3), while the remaining two (M2 and M4) are five-phase. Such a situation will exist always when the even phase number \(n\) is such that \(n/2\) is a prime number, as discussed in the next section. Once more, the two machines with the higher phase number have to be connected at first in series with the source. Five-phase machines are then added at the end of the chain, as shown in Fig. 3.
In the three cases illustrated so far it was possible to connect the maximum possible number \( k = (n-2)/2 \) of machines. Indeed, for any even phase number one expects, on the basis of the considerations given so far, that the number of connectable machines will be \( k = (n-2)/2 \). This is however not always the case. Consider for example a twelve-phase system. Connectivity matrix is shown in Table IV. Machines M1 and M5 are twelve-phase, machine M2 is six-phase, machine M3 is four-phase, and machine M4 is three-phase. Hence, it is not possible to connect all five machines in series since the ratio 4/3 is not an integer (an attempt to connect a four-phase machine to a three-phase machine leads to short-circuiting of all the terminals). At most four machines can be connected in series, two twelve-phase, followed by the six-phase and three-phase. The ordering is M1, M5, M2, M4.

Table V summarises the situation which arises for all the even phase numbers up to eighteen. Bold boxes apply to the phase numbers such that \( n/2 \) is a prime number. As can be seen from the table, only \( k/2 \) machines are \( n \)-phase, while the remaining \( k/2 \) machines are \( n/2 \)-phase. In the eighteen-phase case the number of connectable machines is smaller than eight (at most seven), since the ratio 9/6 is not an integer. Similar situation arises in the twenty-phase case (Table VI), where both four-phase and five-phase machines appear. There are five twenty-phase machines (M1,M3,M7,M9), two ten-phase machines (M2,M6), two five-phase machines (M4,M8) and one four-phase machine (M5). At most eight machines can be connected using the sequence M1,M3,M7,M9,M2,M6,M4,M8 (four twenty-phase, two ten-phase, and two five-phase). But, eight machines can be connected in series with the odd number of phases \( n = 17 \), which requires three inverter legs less.

4. NUMBER OF CONNECTABLE MACHINES

There are three different situations that may arise, depending on the properties of the phase number \( n \). At least one case for each of them has been illustrated in the previous section.

a) Let the number \( n/2 \) be a prime number. The number of machines that can be connected in series equals

\[
k = (n-2)/2
\]

(6)

The number of phases of individual \( k \) machines will be as follows: \( k/2 \) machines will be \( n \)-phase and \( k/2 \) machines will be \( n/2 \)-phase. The ordering of machines has to follow the rule that higher number of phases comes first. Hence the first \( k/2 \) machines are \( n \)-phase, while the subsequent \( k/2 \) machines are \( n/2 \)-phase. This means that one half of the machines are with an even number of phase, while the rest are with an odd number of phases. The phase numbers belonging to this category are \( n = 6, 10, 14, 22, 26, 34, 38, 46, 58, 62, 74 \) etc. The six-phase and the ten-phase case were elaborated in the previous section.

b) Consider next the number of phases \( n \) such that \( n/2 \) is not a prime number, but it satisfies the condition

\[
n = 2^m, \quad m = 3, 4, 5, \ldots.
\]

(7)

The number of machines that can be connected remains to be given with (6). Once again not all \( k \) machines are of the phase number equal to \( n \). In the example illustrated previously \( n = 8, k = 3 \). However, only two machines are eight-phase, while the third one is four-phase. Hence for the general case of \( m \geq 3 \) the phase numbers of the machines that can be connected in series will be:

\[
n, \quad n/2, \quad n/2^2, \quad \ldots, \quad n/2^{m-2}
\]

(8)

This case arises when the phase number of the multi-drive system takes values of \( n = 8, 16, 32, 64 \), etc.

c) The third possible case arises for all the other even \( n \). The number of machines that can be connected is
Series connection of machines whose phase number ratio is not an integer is not possible. This case was illustrated for \( n = 12 \) and 20 in the previous section. Again, among these \( k \) machines only a certain number is with \( n \) phases. The other machines have phase numbers equal to \( n/2, n/3, n/4... \) as appropriate. There are at least three different phase numbers among the multi-machine set. Phase numbers that belong to this group are 12, 18, 20, 24, 28, etc.

Cases b) and c) can be regarded as sub-cases of a more general case for which \( n/2 \) is not a prime number. The total number of connectable machines is however not the same, ((6) and (9), respectively). A summary of all possible situations for an even number of phases is provided in Table VII. Of practical value are only phase numbers that enable series connection of the maximum number of connectable machines (i.e. cases a) and b)).

5. VECTOR CONTROL OF THE MULTI-MOTOR SYSTEM

A standard method of achieving indirect rotor flux oriented control of a current-fed ac machine is considered here. It is assumed that there is a rotor position sensor attached to each machine of the group. The basic form of the vector controller is the same as for a three-phase machine of the same type and the only difference is in the co-ordinate transformation, where \( n \) phase current references are generated by means of the co-ordinate transformation described with (1) and (2), instead of three. The indirect vector controller for operation in the base speed (constant flux) region is illustrated in Fig. 4 for an \( n \)-phase induction machine. The form of the vector controller is the same for a permanent magnet synchronous and synchronous reluctance motor, provided that the constant \( K_1 \) is set to zero and that an appropriate value is assigned to the stator d-axis current reference.

Current control is performed in the stationary reference frame, using inverter phase currents. It is important to notice that the presented form of the vector-controlled multi-phase multi-motor drive system is valid when the current control is exercised in the stationary reference frame. This is so since minimisation of the inverter phase current errors through inverter switching automatically generates appropriate voltages required for compensation of the additional voltage drops in the machines, caused by the flow of x-y current components. The concept can be extended to current control in the rotating reference frame. This is however beyond the scope of this paper.

Either ramp-comparison or hysteresis current control can be used. Generation of individual machine phase current references is done first, using Fig. 4 (superscript \( Mj \) stands for the machine under consideration, \( M1 \) to \( Mk \)):

\[
\begin{align*}
    i_1^{(Mj)} & = \frac{2}{n} \left[ i_{ds}^{(Mj)} \cos \phi_r^{(Mj)} - i_{qs}^{(Mj)} \sin \phi_r^{(Mj)} \right] \\
    i_2^{(Mj)} & = \frac{2}{n} \left[ i_{ds}^{(Mj)} \cos \left( \phi_r^{(Mj)} - \alpha \right) - i_{qs}^{(Mj)} \sin \left( \phi_r^{(Mj)} - \alpha \right) \right] \\
    i_n^{(Mj)} & = \frac{2}{n} \left[ i_{ds}^{(Mj)} \cos \left( \phi_r^{(Mj)} - (n-1)\alpha \right) - i_{qs}^{(Mj)} \sin \left( \phi_r^{(Mj)} - (n-1)\alpha \right) \right]
\end{align*}
\]  

(10)

Inverter reference currents are further built, respecting the appropriate connection diagram for the given number of inverter phases. Inverter reference current creation has to take into account the existence of machines with different phase numbers within the group. Taking as an example the six-phase inverter with a six-phase and a three-phase machine, illustrated in Fig. 1 and Table I, the inverter current references are determined with:
Similarly, for the ten-phase system, illustrated in Fig. 3 and Table III, the inverter phase current references are governed with the following expressions:

\[
\begin{align*}
   i_A^* &= i_{a1} + 0.5i_{a2} + 0.5i_{a3} + 0.5i_{a4} \\
   i_B^* &= i_{b1} + 0.5i_{b2} + 0.5i_{b3} + 0.5i_{b4} \\
   i_C^* &= i_{c1} + 0.5i_{c2} + 0.5i_{c3} + 0.5i_{c4} \\
   i_D^* &= i_{d1} + 0.5i_{d2} + 0.5i_{d3} + 0.5i_{d4} \\
   i_E^* &= i_{e1} + 0.5i_{e2} + 0.5i_{e3} + 0.5i_{e4}
\end{align*}
\]

Similarly, for the ten-phase system, illustrated in Fig. 3 and Table III, the inverter phase current references are governed with the following expressions:

\[
\begin{align*}
   i_A^* &= i_{f1} + i_{f2} + 0.5i_{f3} + 0.5i_{f4} \\
   i_B^* &= i_{f1} + i_{f2} + 0.5i_{f3} + 0.5i_{f4} \\
   i_C^* &= i_{g1} + i_{g2} + 0.5i_{g3} + 0.5i_{g4} \\
   i_D^* &= i_{h1} + i_{h2} + 0.5i_{h3} + 0.5i_{h4} \\
   i_E^* &= i_{j1} + i_{j2} + 0.5i_{j3} + 0.5i_{j4}
\end{align*}
\]

Inverter phase current references will be built in this manner for any phase number \( n \). It is only necessary to form an appropriate connectivity matrix and the corresponding connection diagram in order to arrive at an equation of the form given in (11) and (12). Since current control is performed in the stationary reference frame using total inverter phase currents, inverter output phase voltages will be of the form required to minimize the phase current errors. Inverter output phase voltages are again governed with the appropriate connection diagram for the given number of phases. For the already considered six-phase case the inverter output phase voltages follow directly from Fig. 1:

\[
\begin{align*}
   v_A &= v_{a1} + v_{a2} \\
   v_B &= v_{b1} + v_{b2} \\
   v_C &= v_{c1} + v_{c2} \\
   v_D &= v_{d1} + v_{d2} \\
   v_E &= v_{e1} + v_{e2} \\
   v_F &= v_{f1} + v_{f2} + v_{f3} + v_{f4}
\end{align*}
\]

Similarly, for the ten-phase inverter the voltages are determined with the connection diagram of Fig. 3:

\[
\begin{align*}
   v_A &= v_{a1} + v_{a2} + v_{a3} + v_{a4} \\
   v_B &= v_{b1} + v_{b2} + v_{b3} + v_{b4} \\
   v_C &= v_{c1} + v_{c2} + v_{c3} + v_{c4} \\
   v_D &= v_{d1} + v_{d2} + v_{d3} + v_{d4} \\
   v_E &= v_{e1} + v_{e2} + v_{e3} + v_{e4} \\
   v_F &= v_{f1} + v_{f2} + v_{f3} + v_{f4} + v_{a1} + v_{a2} + v_{a3} + v_{a4}
\end{align*}
\]

6. SIMULATION VERIFICATION OF THE MULTI-PHASE MULTI-MOTOR SYSTEM

The concept of the multi-phase multi-motor drive system, developed in this paper, is verified by performing simulation of a ten-phase four-motor drive, consisting of four induction motors. Relevant per-phase equivalent circuit parameters and other data of the machines are given in the Appendix (all the four machines are assumed to have the same per-phase equivalent circuit parameters and ratings). The system consists of two ten-phase and two five-phase machines and inverter current reference generation is described with (12). The current controlled PWM inverter is treated as ideal in simulation, so that the inverter phase current references are equated to the inverter output phase currents. Stator currents of all the machines are therefore known and stator phase voltages are obtained by reconstruction. Inverter output phase voltages are then calculated using (14). All the four induction machines are represented for simulation purposes with the appropriate phase variable models. The machine models obtainable using general theory of electrical machines are therefore not utilised and such an approach leads to an ultimate proof of the concept. Torque mode of operation is examined.
Excitation of all the four machines is initiated simultaneously, by ramping the rotor flux reference from zero to twice the rated value in the time interval from zero to 0.01 s. The rotor flux reference is brought back to the rated value (1.797 Wb for the ten-phase and 1.27 Wb for the five-phase machines) in the time interval from 0.05 to 0.06 s in the linear manner and is further kept unchanged. A forced excitation, leading to a faster build-up of the rotor flux in the machines, is obtained in this way. Upon completion of the excitation transient, different torque commands, of differing duration, are applied to the four machines in different time instants. Application and removal of the torque command is in all the cases ramp-like, with the ramp duration of 0.01 s. Rated torque command (16.667 Nm) is applied to the ten-phase machine (IM1) at 0.5 s and is removed at 0.7 s. A torque command equal to 2/3 of the rated torque (i.e. 11.11 Nm) is applied to the second ten-phase machine (IM2) at 0.45 s and removal commences at 0.65 s. Rated torque command (8.33 Nm) is applied to the five-phase machine (IM3) at 0.4 s and the removal is initiated at 0.65 s. Torque command for the second five-phase machine (IM4) is applied at 0.3 and the removal starts at 0.55 s (¾ of the rated torque, i.e. 6.25 Nm). Load torque is zero for all machines.

Figure 5 shows rotor flux reference and the corresponding response of the rotor flux in the four machines. Excitation process of any of the four machines is not disturbed in any way by the presence of the other machines in the group. Rotor flux in any of the machines attains the reference value, confirming the absence of any x-y rotor flux components. Upon completion of the excitation transient rotor flux remains at the constant value equal to the reference, regardless of what happens with any of the four machines further on. Torque reference and torque response (which are indistinguishable one from the other due to assumed ideal current feeding) are shown in Fig. 6. Torque response of any of the four machines is not affected at all by the presence of the other machines. Furthermore, torque control of any of the four machines is completely decoupled from the flux control. Thus not only that full decoupled flux and torque control of any particular machine is achieved, but the completely independent control of all the machines results as well, due to the introduced phase transposition, as predicted by the theoretical considerations. Corresponding speed responses are shown in Fig. 7. They are perfectly smooth and the fastest possible due to the achieved complete decoupling of the torque control of the four machines.

Stator phase current references for the four machines are illustrated by means of Fig. 8, where the traces for phase ‘a’ are shown. They are of familiar waveform, met in the case of any vector controlled three-phase machine, and are sinusoidal in final steady states. However, total inverter current references, shown in Fig. 9 for the first four phases of the inverter, are highly distorted since they are determined with the summation given in (12), which accounts for the phase transposition.

Phase voltages of individual machines are illustrated for phase ‘a’ in Fig. 10. As can be seen from this figure, phase voltages of all the four machines start changing at the instant of the application of the torque command to IM4 (0.3 s). Since IM4 is a five-phase machine, its acceleration affects all the other machines in the group. The flow of x-y stator current components through the windings of all four machines is responsible for this. Waveforms of phase voltages show a certain amount of distortion, caused by the series connection. It should be noted that the two five-phase machines are affecting only one another, while the two ten-phase machines are affected by all the other machines in the group.

Inverter output phase voltages, obtained by means of (14), are displayed in Fig. 11 for the first four phases. They are heavily distorted since they represent an appropriate sum of four individual machine phase voltages, of different fundamental frequencies and already having some amount of distortion due to the x-y components of the machines’ phase voltages. However, generation of such distorted inverter voltage waveforms presents no difficulty
in practice, provided that fast inverter current control with a sufficiently high inverter switching frequency is used. The practical questions of how much the dc link voltage should be, as a function of the machines’ rated voltages, and how much the voltage reserve for good current control needs to be, are left for future investigation.

7. CONCLUSION

The paper develops a novel concept for a multi-motor drive system, which enables independent control of a certain number of multi-phase machines with even numbers of phases, although the whole system is supplied from a single current-controlled voltage source inverter. The stator multi-phase windings have to be connected in series with an appropriate phase transposition, in order to achieve the independent control of the machines in the system. The concept is developed in a systematic manner, using general theory of electrical machines, and is valid regardless of the type of the ac machine, so that different machine types can be used within the same multi-motor drive system. The necessary phase transposition in the stator winding connection is established by analysing the properties of the decoupling transformation matrix and so-called connectivity matrix is formed for selected phase numbers. Corresponding connection diagrams are further developed on the basis of the connectivity matrix. The concept is in general applicable to any phase number greater than or equal to five. Considerations in this paper are restricted to an even phase number. An even number of phases and the previous odd number enable, at best, connection of the same number of machines in series and therefore the odd phase number saves a larger number of inverter legs, compared to an equivalent three-phase drive system. However, as shown in the paper, an even phase number always leads to a multi-motor system with at least two different phase numbers. All the machines in the group with a smaller number of phases are not affected by series connection to the machines with a larger phase number. The negative consequences of the series connection, discussed in the next paragraph, are therefore less pronounced for an even phase number than for an odd phase number.

An obvious and major drawback of the concept is an increase in the stator winding losses due to the flow of the flux/torque producing currents of all the machines through stator windings of some of the machines (note that rotor winding losses are not affected). Similarly, stator iron losses will increase as well (although to a much lesser extent), due to the increased phase voltage of the individual machines caused by the flow of x-y current components. This will decrease the efficiency of the affected machines in the multi-motor system and will yield an overall reduction in the total efficiency of the drive system, when compared to an equivalent three-phase counterpart. Reduction in the efficiency is expected to be smaller for a system with an even phase number, when compared to the equivalent multi-motor system with an odd phase number. In the ten-phase case considered in detail, the two five-phase machines only affect one another, and do not suffer from any adverse effects caused by connection to the two ten-phase machines. Of course, the two ten-phase machines are affected by the presence of all the other machines.

It has to be noted that some of the advantages of the multi-phase machines, which exist in the case of a single multi-phase motor drive, have been lost in the proposed multi-motor multi-phase system. For example, torque density cannot be increased by injection of higher stator current harmonics, since all the available degrees of freedom are used to control other machines in the group. Similarly, fault tolerance is completely lost for the same reason.
All the possible even phase numbers are examined (disregarding the physical feasibility of large phase numbers) in order to establish which of them offer a potential for connection of the largest number of machines. It is shown that the maximum number of connectable machines results when the phase number \( n \) is either such that \( n/2 \) is a prime number or is a power of two.

The concept is verified by simulation of a four-motor ten-phase drive system. Torque mode of operation is examined and it is shown that completely decoupled and independent vector control of the four machines is possible with the proposed series connection. The major advantage of such a multi-drive system is the saving in the required number of inverter legs (when compared to an equivalent multi-motor three-phase drive), thus leading to an increase in reliability. The ten-phase system requires ten inverter legs, while the corresponding four-motor three-phase system would ask for twelve inverter legs. The other extremely useful feature of the concept is the easiness of implementation within a single DSP. It is necessary to execute the required number of vector control algorithms in parallel, calculate the individual motor phase current references, and then give at the output of the DSP inverter current references, obtained by summation and respecting the connection diagram. Current control is further executed in the stationary reference frame, using either hysteresis or ramp comparison inverter phase current control.

8. ACKNOWLEDGEMENT

The authors gratefully acknowledge support provided for the work on this project by the EPSRC, under the standard research grant number GR/R64452/01, and by Semikron Ltd. Mr. M. Jones acknowledges financial support provided for his PhD studies by the IEE, through the IEE Robinson Research Scholarship.
9. REFERENCES


10. APPENDIX

Per-phase equivalent circuit parameters of the 50 Hz five-phase and ten-phase induction motors:

\[ R_s = 10 \Omega \quad R_r = 6.3 \Omega \]
\[ L_{ds} = L_{dr} = 0.04 \text{ H} \quad L_m = 0.42 \text{ H} \]

Inertia and number of pole pairs: \( J = 0.03 \text{ kg m}^2 \), \( P = 2 \). Rated per-phase torque: 1.667 Nm. Rated rotor flux (RMS): 0.568 Wb. Rated per-phase voltage and current (RMS): 220 V and 2.1 A.

11. LIST OF PRINCIPAL SYMBOLS

- \( v, i, \psi \) - voltage, current and flux linkage, respectively
- \( \theta_s, \theta \) - transformation angle for stator variables and rotor instantaneous angular position, respectively
- \( \omega_s, \omega \) - angular speed of the common reference frame and angular electrical speed of rotor, respectively
- \( R_s, R_r \) - stator and rotor per-phase resistance
- \( L_{ds}, L_{dr}, L_m \) - stator and rotor per-phase leakage inductance, magnetising inductance
- \( P, T_e \) - number of pole pairs and electromagnetic torque of the machine, respectively
- \( p \) - Laplace operator
- \( \phi_r \) - instantaneous position of the rotor flux

Indices:
- \( s, r \) - stator and rotor, respectively
- \( d, q \) - \( d-q \) axis components of voltages, currents and flux linkages in the common reference frame
- \( x, y \) - \( x-y \) components of voltages, currents and flux linkages
- \( o^+, o^- \) - zero sequence components of voltages, currents and flux linkages
- \( n \) - rated value
TABLES WITH NUMBERS AND TITLES

Table I. Connectivity matrix for the six-phase drive system.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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Table II. Connectivity matrix for the eight-phase drive system.

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Table III. Connectivity matrix for the ten-phase drive system.

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Table IV. Connectivity matrix for the twelve-phase system.

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Table V. Phase numbers of individual machines for the supply phase numbers up to eighteen.

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<th>Number of the supply phases</th>
<th>6</th>
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<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
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<td>Number of phases and ordering of connectable machines (before re-ordering)</td>
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<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
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<td>14</td>
<td>16</td>
<td>18</td>
<td>7</td>
<td>8</td>
<td>3</td>
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</tbody>
</table>
**Table VI.** Connectivity matrix for the twenty-phase drive system.

|   | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
| M1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| M2 | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 |
| M3 | 1 | 4 | 7 | 10 | 13 | 16 | 19 | 2 | 5 | 8 | 11 | 14 | 17 | 20 | 3 | 6 | 9 | 12 | 15 | 18 |
| M4 | 1 | 5 | 9 | 13 | 17 | 1 | 5 | 9 | 13 | 17 | 1 | 5 | 9 | 13 | 17 | 1 | 5 | 9 | 13 | 17 |
| M5 | 1 | 6 | 11 | 16 | 1 | 6 | 11 | 16 | 1 | 6 | 11 | 16 | 1 | 6 | 11 | 16 | 1 | 6 | 11 | 16 |
| M6 | 1 | 7 | 13 | 19 | 5 | 11 | 17 | 3 | 9 | 15 | 1 | 7 | 13 | 19 | 5 | 11 | 17 | 3 | 9 | 15 |
| M7 | 1 | 8 | 15 | 2 | 9 | 16 | 3 | 10 | 17 | 4 | 11 | 18 | 5 | 12 | 19 | 6 | 13 | 20 | 7 | 14 |
| M8 | 1 | 9 | 17 | 5 | 13 | 1 | 9 | 17 | 5 | 13 | 1 | 9 | 17 | 3 | 11 | 17 | 5 | 13 | 1 | 9 |
| M9 | 1 | 10 | 19 | 8 | 17 | 6 | 15 | 4 | 13 | 2 | 11 | 20 | 9 | 18 | 7 | 16 | 5 | 14 | 3 | 12 |

**Table VII.** Number of connectable machines and their phase order for an even system phase number.

<table>
<thead>
<tr>
<th>$n = \text{an even number, } \geq 6$</th>
<th>Number of connectable machines</th>
<th>Number of phases of machines</th>
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</thead>
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<td>$n/2 = \text{prime number}$</td>
<td>$k = \frac{n-2}{2}$</td>
<td>$k/2 \text{ are } n\text{-phase and}$</td>
</tr>
<tr>
<td>$n/2 \neq \text{prime number}$</td>
<td>$k = \frac{n-2}{2}$</td>
<td>$k/2 \text{ are } n/2\text{-phase}$</td>
</tr>
<tr>
<td>$n = 2^m, m = 3, 4, 5, \ldots$</td>
<td>$n, \frac{n}{2}, \frac{n}{2^2}, \ldots, \frac{n}{2^{m-2}}$</td>
<td></td>
</tr>
<tr>
<td>all other even numbers</td>
<td>$k &lt; \frac{n-2}{2}$</td>
<td>$n, \frac{n}{2}, \frac{n}{3}, \frac{n}{4}, \ldots$ as appropriate</td>
</tr>
</tbody>
</table>
LIST OF FIGURE CAPTIONS

Fig. 1. Connection diagram for the six-phase two-motor system.

Fig. 2. Connection diagram for the eight-phase drive system.

Fig. 3. Series connection of two ten-phase and two five-phase machines to the ten-phase source.

Fig. 4. Indirect vector controller for an $n$-phase induction motor ($K_r = 1/(T_r i_{dr})$).

Fig. 5. Rotor flux reference and rotor flux response of the four machines in the ten-phase drive system.

Fig. 6. Torque references and torque responses of the four machines.

Fig. 7. Speed responses of the four-motor drive system.

Fig. 8. Phase ‘a’ current references of the four machines.

Fig. 9. Inverter phase current references for the first four phases.

Fig. 10. Stator phase ‘a’ voltages of the four machines.

Fig. 11. Inverter output phase voltages for the first four phases.
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Fig. 10. Stator phase ‘a’ voltages of the four machines.
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